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Abstract : In this thesis, we study some problems bound to the theory of deformation of the discontinuous groups on solvable homogeneous spaces, namely the stability, the rigidity, and some related geometrical problems. Given a locally compact group G, H a closed subgroup of G and Γ a discontinuous group for the homogeneous space G/H, that is Γ is discrete and acts properly and freely on G/H. We designate by $\mathrm{Hom}(\Gamma, G)$, the set of all the homomorphisms of groups from Γ to G endowed with the pointwise convergence topology. The parameter space, noted $R(\Gamma, G, H)$, is defined as follows :

$$R(\Gamma, G, H) = \{\psi \in \mathrm{Hom}(\Gamma, G) : \psi \text{ is injective and } \psi(\Gamma) \text{ is discontinuous for } G/H\}.$$

The deformation space, noted $\tau(\Gamma, G, H)$, is defined as the space of the orbits of the action of G on the parameter space $R(\Gamma, G, H)$ which is defined as follows :

$$g.\psi = g\psi g^{-1},\ \forall g \in G, \forall \psi \in R(\Gamma, G, H).$$

We say that an element $\psi \in R(\Gamma, G, H)$ is stable if there is a neighborhood of ψ in the topological space $\mathrm{Hom}(\Gamma, G)$ which is included in $R(\Gamma, G, H)$. ψ is said to be locally rigid, if the orbit $G \cdot \psi$ is open in $R(\Gamma, G, H)$. My work on this subject concerns some classes of connected nilpotent Lie groups. We handled the following situations :

- G is the reduced Heisenberg Lie group.
- G is the product of reduced Heisenberg Lie groups and H is the diagonal subgroup of G.
- G is the reduced threadlike Lie group and Γ is a non-abelian subgroup of G.

We show that the stability property holds for the classes above. As for the rigidity, we gave an affirmative answer to the following conjecture substantiated by A. Baklouti :

Conjecture : *Let G be a connected nilpotent Lie group, H a connected subgroup of G and Γ a discontinuous group for G/H. Then, the local rigidity holds if and only if Γ is a finite group.*

For the first two classes above, we also show that the Hausdorffness property of the deformation space holds, which is endowed with smooth manifold structure in the case where G is the reduced Heisenberg Lie group. For the second class, we proved that the deformation space $\tau(\Gamma, G, H)$ is a Hausdorff space if and only if Γ is finite.